Non-Hermitian management of light in multimode fibers
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Abstract
We show that the simultaneous modulation of the propagation constant and of the gain/loss along the multimode fibers results in unidirectional coupling among the modes, which, depending on the modulation parameters, leads to the enhancement or reduction of the excitation of higher order transverse modes. In the latter case the mode-cleaning is observed, in ideal case resulting in single-mode spatially coherent output. On the contrary, coupling towards higher order modes may enhance pulsing, turbulence and, eventually help in super-continuum generation.

Motivation

- Hermitian potential: index modulation
- Bidirectional, symmetric, mode coupling
- Couples any mode \( k_q \) with \( k_q + q \) and \( k_q - q \)

\[
V(z) = \cos(qz) = \frac{1}{2} \left( e^{i \phi} + e^{-i \phi} \right)
\]

\[
A(k) = \frac{1}{2} \left( e^{i \phi} + e^{-i \phi} \right)
\]

- Non-Hermitian potential: index and gain/loss modulation
- Unidirectional, asymmetric, mode coupling
- Couples any mode \( k_q \) with \( k_q + q \) only

\[
V(z) = \cos(qz) - i \sin(qz) = \exp(-i qz)
\]

Simplified model for Gaussian solution

\[
\frac{d^2 \beta}{dz^2} = 8d \beta \rho e^{-\rho \beta} - \frac{m_2}{k} \cos(qz + \phi)
\]

\[
\frac{d \beta}{dz} = 4d \beta \rho e^{-\rho \beta} + b \frac{m_1}{k} \cos(qz - 4d \beta e)
\]

\[
\frac{d \rho}{dz} = 8d \beta \rho e^{-\rho \beta} - 2m_2 \cos(qz + \phi) \rho
\]

Full Model: (2+1)D Linear Schrödinger Equation

\[
\frac{\partial A}{\partial z} = \frac{i}{2} \left( \frac{\partial^2 A}{\partial r^2} + \frac{\partial^2 A}{\partial z^2} + i \left[ m_1 \cos(qz) + i m_2 \sin(qz + \frac{\pi}{2}) \right] \right) e^{-i qz} A
\]

Results

Drift and Amplitude of oscillations in \((\phi, \theta)\) space

\[
V(z) = [m_1 \cos(qz) + im_2 \cos(qz + \phi)]
\]

\[
m_1 = m \cos(\theta), m_2 = m \sin(\theta)
\]

\[
m = 8 \times 10^{-4}, \quad m = 0.95 + \Delta m
\]

\[
\Delta m = 0.0088
\]

Evolution in phase \((\beta_n, \beta_{cm}, \rho)\)

 Dependency on modulation frequency \((q)\)

\[
q_{res} = \frac{2 \sqrt{A}}{r_G}
\]

Non-Hermitian Beam cleaning using the Full model

Higher order Laguerre-Gaussian mode generation

\[
\psi = \frac{1}{\sqrt{\pi}} \int \left[ \int \left| A \right|^2 \, dx \, dy \right] \left[ \int \left| \psi \right|^2 \, dx \, dy \right] \, dx \, dy
\]

\[
U \text{ is the total field}
\]

Conclusions

- Powerful tool to control the dynamics of the propagating light, coupling towards higher or lower order modes is governed by the relative phase \((\phi)\).
- Propagation of speckle beam along this modulated fiber, improvement of the spatial structure of the speckle beam along the propagation.
- Coupling towards higher order modes leads to the generation of higher order modes.

References