Turbulence control
by non-Hermitian Potentials

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abstract

We propose a method for a smart control of turbulent states by modifying the excitation energy cascade of turbulence. The method is based on the asymmetric coupling between the spatial excitation modes by non-Hermitian potentials [1]. The unidirectional coupling towards larger (smaller) wavenumbers increases (reduces) the energy flow to turbulent states, and thus influence the character of turbulence. The study is based on the Complex Ginzburg-Landau Equation which is an universal model for pattern formation and turbulence in many different systems [2]. We show that the enhancement or reduction of the turbulence is indeed governed by the introduced direction of the energy flow, depending on the phase shift between the real and imaginary parts of the temporal oscillation of the non-Hermitian potential.

motivation

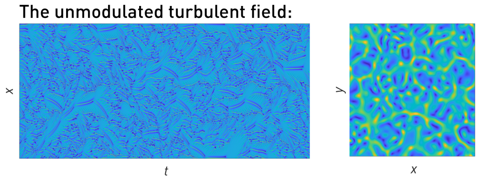
The model:

$$\frac{\partial A}{\partial t} = (1 - i\alpha)\left(1 - |A|^2\right)A + (i + d)\nabla^2 A + iV(r,t)A$$

Travelling wave solution

$$A(x,t) = \left(1 - dk^2\right)^{1/2} e^{ikx+\omega t}$$

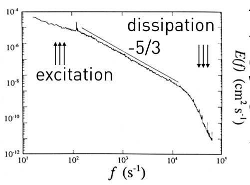
The unmodulated turbulent field:



General scheme of excitation energy cascade of turbulence

Da Vinci drawings

Kolmogorov law:



Turbulence cascade bridges these two exitation and dissipation scales.
We want to affect the cascade of turbulence

figure from [3]

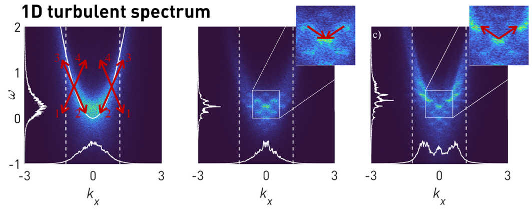
The potential:

$$V(x,t) = \cos(qx)\, e^{\pm i\Omega t}$$

- + couples ω to ω+Ω (3 and 4)
- − couples ω to ω−Ω (1 and 2)

symmetrically couples *k* with modes *k+q* and *k−q*

1D turbulent spectrum



results

The general potential:

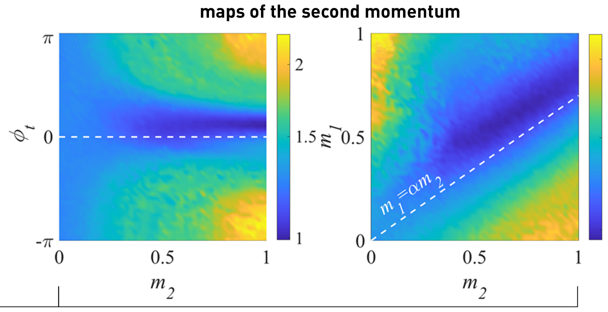
$$V(x,t) = \cos(qx)\left[m_1\cos(\Omega t) + i m_2\cos(\Omega t + \phi_t)\right]$$

Analytical analysis conclude that the desired unidirectional coupling happens for Φ_t=0 and $m_1/m_2=\alpha$

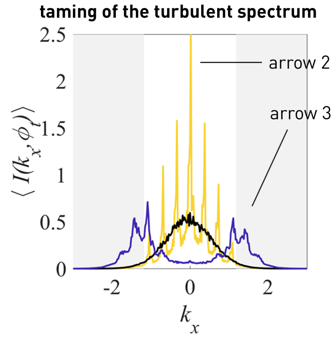
Second order momentum of the 2D Fourier spectrum:

$$\mu = \frac{\int_{-\infty}^{\infty} k_x^2 \int_{-\infty}^{\infty} \left|A(k_x,\omega)\right|^2 d\omega dk_x}{\int_{-\infty}^{\infty} \left|A(k_x,\omega)\right|^2 d\omega dk_x}$$

maps of the second momentum



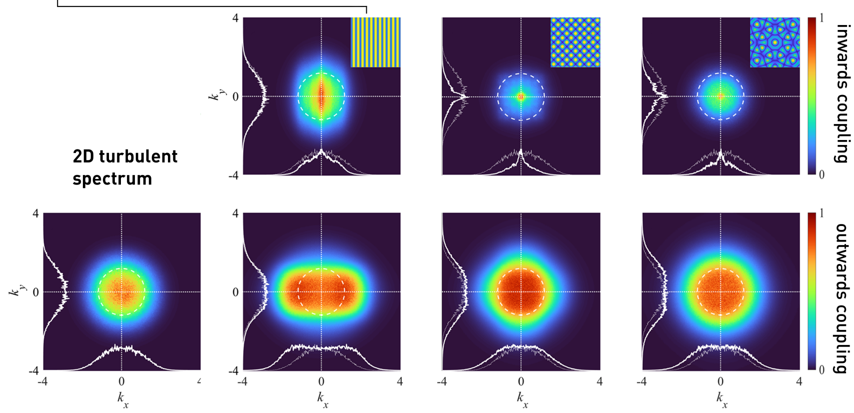
taming of the turbulent spectrum



Generalisation to 2D

$$V(x,t) = V(\mathbf{r})\left[m_1\cos(\Omega t) + m_2\cos(\Omega t + \phi_t)\right]$$

2D turbulent spectrum



conclusions

- Non-Hermitian potentials periodically modulated in space and time can inlfuence the exitation cascade mechanism in turbulence.
- We are able to regularise the spectrum and oppositely, enhance turbulence by shifting the real and imaginary part of the temporal modulation.
- We prove this idea on the Ginzburg-Landau equation for 1D and 2D systems.
- The proposed turbulence control method could be implemented in different systems for being proved in a universal model.

references

[1] El-Ganainy, Ramy, et al. "Non-Hermitian physics and PT symmetry." Nature Physics 14.1 (2018): 11-19.
[2] Aranson, Igor S., and Lorenz Kramer. "The world of the complex Ginzburg-Landau equation." Reviews of modern physics 74.1 (2002): 99.
[3] Zocchi, Giovanni, et al. "Measurement of the scaling of the dissipation at high Reynolds numbers."Physical Review E 50.5 (1994): 3693.